Power Correlation Coefficient of a General Fading Model

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***Abstract*— This work provides the power correlation coefficient of a very general fading model. The model considers the signal as composed of identically/non-identically distributed clusters, allows for stationary/nonstationary environments, and includes many distributions as particular cases, such as the Rayleigh, Rice, Nakagami-m, and Hoyt (Nakagami-q) ones. After deriving the power correlation coefficient of the general model, that statistic is particularized for some important simplified models. Finally, for the Rayleigh, Rice, and Hoyt distributions, the power correlation coefficient is investigated in both space domain and frequency domain.**

***Index Terms*— Correlation coefficient, fading model, Hoyt distribution, Nakagami-m distribution, Rayleigh distribution, Rice distribution.**

I. Introduction

In wireless communications, the signal strength fluctuates randomly throughout the propagation environment in a fast fading condition. This fluctuation is caused by the multipath phenomenon, in which the signal reaching the receiver is composed of a large number of scattered waves that arise from different paths. The amplitudes and phases of these waves are random variables, and hence the envelope and the phase of the received signal have random behaviors. In some physical configurations, besides the scattered waves, the signal is also influenced by a specular (or dominant) wave.

Many distributions have already been proposed to describe the signal envelope in a fading channel. In some of them, such as the Rayleigh [1], the Rice [2], [3], the Hoyt (Nakagami-q) [4], the Asymmetrical $\eta-\kappa$ [5], the Symmetrical $\eta-\kappa$ [6], and the Generalized $\eta-\kappa$ distributions, the in-phase and quadrature signal components are Gaussian variates. The difference among these models are in the assumptions concerning the means and variances of the quadrature components: the Rayleigh model considers zero means and equal variances; the Ricean model, arbitrary means and equal variances; the Hoyt model, zero means and arbitrary variances; the Asymmetrical $\eta-\kappa$ model, zero mean in one quadrature component, arbitrary mean in the other, and arbitrary variances; the Symmetrical $\eta-\kappa$ model, arbitrary means and variances whose ratio is equal to the ratio of the squared means; and the Generalized $\eta-\kappa$ model, arbitrary means and variances. In other distributions, such as the Nakagami-m [7], the $\kappa-\mu$ [8], and the $\eta-\mu$ [9], the signal is composed of clusters of waves. These models differ from each other in the statistical properties of the clusters: in the Nakagami-m model, each cluster follows the Rayleigh distribution; in the $\kappa-\mu$ model, the Rice distribution; and in the $\eta-\mu$ model, the Hoyt distribution.

In this work, we derive the correlation coefficient of the instantaneous powers (or squared envelopes) of two signals in a very general fading model. Such a model considers each signal as composed of clusters of waves. The in-phase and quadrature components of each cluster are Gaussian random variables with arbitrary means and variances. Thus, the model includes all previously mentioned models as particular cases. Furthermore, the model is valid for both identically and non-identically distributed clusters, and for both stationary[1] and nonstationary environments.

After obtaining the power correlation coefficient of the general fading model, we particularize this statistic for some important models. Finally, numerical results are presented in order to compare the power correlation coefficient of the Rayleigh, Rice, and Hoyt distributions in both space domain and frequency domain.

The paper is organized as follows. In Section II, the general fading model is described. In Section III, the power correlation coefficient is derived. In Section IV, some plots and results are examined. In Section V, some conclusions are drawn.

II. General Fading Model

In this section, we describe the marginal and joint statistics concerning two signals, S_1 and S_2, of the general fading model investigated in this work.

A. Marginal Statistics

The signal of our general fading model consists of clusters of waves, each of which composed of a dominant wave and of multipath (or scattered) waves. Denoting the envelope of the signal S_i as R_i, the corresponding instantaneous power W_i (or squared envelope R_i^2) is

$$W_i = R_i^2 = \sum_{j=1}^{n_i} R_{i,j}^2 \qquad i = 1, 2 \tag{1}$$

where n_i is the number of clusters of S_i. In each signal, the variates $R_{i,j}$, $j = 1, 2, ..., n_i$, are mutually independent and given by

$$R_{i,j}^2 = X_{i,j}^2 + Y_{i,j}^2 \qquad j = 1, 2, ..., n_i \tag{2}$$

where $X_{i,j}$ and $Y_{i,j}$ are uncorrelated Gaussian random variables that correspond, respectively, the in-phase and quadrature

[1] In this work, the term *stationary environment* designates the environment where the statistics of one signal are equal to their counterpart of the other signal.

components of each cluster. The variates $X_{i,j}$ and $Y_{i,j}$ have means $m_{X_{i,j}}$ and $m_{Y_{i,j}}$, which arise from the dominant wave of the jth cluster, and variances $\sigma^2_{X_{i,j}}$ and $\sigma^2_{Y_{i,j}}$, which stem from the multipath waves of the jth cluster. Finally, without loss of generality, we shall consider

$$n_2 \geq n_1 \tag{3}$$

In particular, for stationary environments, the statistics of one signal are equal to their counterpart of the other signal. Therefore, in these environments, $n_1 = n_2 = n$ and

$$m_{X_{1,j}} = m_{X_{2,j}} \qquad m_{Y_{1,j}} = m_{Y_{2,j}} \qquad j = 1, ..., n \tag{4a}$$

$$\sigma_{X_{1,j}} = \sigma_{X_{2,j}} \qquad \sigma_{Y_{1,j}} = \sigma_{Y_{2,j}} \qquad j = 1, ..., n \tag{4b}$$

B. Joint Statistics

The dependency of the signals S_1 and S_2 occurs by means of n_0 clusters, $n_0 \leq n_1$. These clusters will be referred to as *shared* clusters and will be indexed, without loss of generality, as the first n_0 clusters of S_1 as well as the first n_0 clusters of S_2. Thus, for $n_0 < j \leq n_i$, $i = 1, 2$, the jth cluster of S_i has no corresponding cluster in the other signal. These clusters will be referred to as *unshared* clusters.

1) Shared Clusters: In the jth *shared* cluster, $X_{1,j}$, $Y_{1,j}$, $X_{2,j}$, and $Y_{2,j}$ are jointly Gaussian random variables, and their correlation coefficients are defined as

$$\begin{aligned}\nu_{1,j} &\triangleq \frac{E\{X_{1,j}X_{2,j}\} - m_{X_{1,j}}m_{X_{2,j}}}{\sigma_{X_{1,j}}\sigma_{X_{2,j}}} \\ &\triangleq \frac{E\{Y_{1,j}Y_{2,j}\} - m_{Y_{1,j}}m_{Y_{2,j}}}{\sigma_{Y_{1,j}}\sigma_{Y_{2,j}}}\end{aligned} \tag{5a}$$

$$\begin{aligned}\nu_{2,j} &\triangleq \frac{E\{X_{1,j}Y_{2,j}\} - m_{X_{1,j}}m_{Y_{2,j}}}{\sigma_{X_{1,j}}\sigma_{Y_{2,j}}} \\ &\triangleq -\frac{E\{Y_{1,j}X_{2,j}\} - m_{Y_{1,j}}m_{X_{2,j}}}{\sigma_{Y_{1,j}}\sigma_{X_{2,j}}}\end{aligned} \tag{5b}$$

where $j = 1, 2, ..., n_0$. In a fading environment, these coefficients depend on the distance between the reception points, on the frequency difference between the transmitted signals, and on the statistical behavior of the angles of arrival and times of arrival of the scattered waves [10], [11].

2) Unshared Clusters: Each one of the *unshared* clusters is present in only one signal. Thus, for $n_0 < j \leq n_1$, the jth cluster of S_1 is statistically independent of the jth cluster of S_2. Consequently, for $n_0 < j \leq n_1$, $X_{1,j}$ and $Y_{1,j}$ are independent of $X_{2,j}$ and $Y_{2,j}$, as well as $R_{1,j}$ is independent of $R_{2,j}$.

3) Distinct Indexes: For distinct indexes, $\forall j \neq k$, $X_{1,j}$ and $Y_{1,j}$ are independent of $X_{2,k}$ and $Y_{2,k}$; hence, $\forall j \neq k$, $R_{1,j}$ and $R_{2,k}$ are independent variates.

III. Power Correlation Coefficient

In this section, we first present some Gaussian statistical properties, which will be useful in calculating the power correlation coefficient. We then derive this statistic for the general fading model described in Section II. Finally, special cases of the general model are considered.

A. Gaussian Statistical Properties

Suppose two jointly Gaussian random variables Z_1 and Z_2, respectively with means m_{Z_1} and m_{Z_2}, variances $\sigma^2_{Z_1}$ and $\sigma^2_{Z_2}$, and correlation coefficient ν. Then, their joint probability density function is

$$\begin{aligned}f_{Z_1Z_2}(z_1, z_2) = &\frac{1}{2\pi\sigma_{Z_1}\sigma_{Z_2}\sqrt{1-\nu^2}} \exp\left[-\frac{(z_1 - m_{Z_1})^2}{2\sigma^2_{Z_1}(1-\nu^2)}\right] \\ &\times \exp\left[\nu\frac{(z_1 - m_{Z_1})(z_2 - m_{Z_2})}{\sigma_{Z_1}\sigma_{Z_2}(1-\nu^2)} - \frac{(z_2 - m_{Z_2})^2}{2\sigma^2_{Z_2}(1-\nu^2)}\right]\end{aligned} \tag{6}$$

By definition, the joint moment $E\{Z_1^2 Z_2^2\}$ is

$$E\{Z_1^2 Z_2^2\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z_1^2 z_2^2 f_{Z_1Z_2}(z_1, z_2)dz_1dz_2 \tag{7}$$

Substituting (6) into (7) and solving the integration yields

$$\begin{aligned}E\{Z_1^2 Z_2^2\} = &4\nu m_{Z_1}m_{Z_2}\sigma_{Z_1}\sigma_{Z_2} + 2\nu^2\sigma^2_{Z_1}\sigma^2_{Z_2} \\ &+ \left(m^2_{Z_1} + \sigma^2_{Z_1}\right)\left(m^2_{Z_2} + \sigma^2_{Z_2}\right)\end{aligned} \tag{8}$$

From the definition of variance

$$E\{Z_i^2\} = m^2_{Z_i} + \sigma^2_{Z_i} \qquad i = 1, 2 \tag{9}$$

and from the definition of covariance

$$Cov\{Z_1^2, Z_2^2\} = E\{Z_1^2 Z_2^2\} - E\{Z_1^2\}E\{Z_2^2\} \tag{10}$$

Replacing (8) and (9) into (10) results

$$Cov\{Z_1^2, Z_2^2\} = 4\nu m_{Z_1}m_{Z_2}\sigma_{Z_1}\sigma_{Z_2} + 2\nu^2\sigma^2_{Z_1}\sigma^2_{Z_2} \tag{11}$$

When $Z_1 = Z_2 = Z$, i.e., $m_{Z_1} = m_{Z_2} = m_Z$, $\sigma_{Z_1} = \sigma_{Z_2} = \sigma_Z$, and $\nu = 1$, the expression (11) reduces to

$$Var\{Z^2\} = 4m_Z^2\sigma_Z^2 + 2\sigma_Z^4 \tag{12}$$

B. General Model

From the independence of $R_{i,j}$ and $R_{i,k}$, $\forall j \neq k$,

$$Var\{W_i\} = \sum_{j=1}^{n_i} Var\{R^2_{i,j}\} \qquad i = 1, 2 \tag{13}$$

Moreover, because $X_{i,j}$ and $Y_{i,j}$ are uncorrelated variates

$$Var\{R^2_{i,j}\} = Var\{X^2_{i,j}\} + Var\{Y^2_{i,j}\} \qquad i = 1, 2 \tag{14}$$

where $j = 1, 2, ..., n_i$.

Applying (12) to $X_{i,j}$ and to $Y_{i,j}$, and using (13) and (14)

$$\begin{aligned}Var\{W_i\} = \sum_{j=1}^{n_i}\Big[&4(m^2_{X_{i,j}}\sigma^2_{X_{i,j}} + m^2_{Y_{i,j}}\sigma^2_{Y_{i,j}}) \\ &+2(\sigma^4_{X_{i,j}} + \sigma^4_{Y_{i,j}})\Big] \qquad i = 1, 2\end{aligned} \tag{15}$$

From the independence of $R_{1,j}$ and $R_{2,k}$, $\forall j \neq k$,

$$Cov\{W_1, W_2\} = \sum_{j=1}^{n_0} Cov\{R^2_{1,j}, R^2_{2,j}\} \tag{16}$$

Of course, the *unshared* clusters have no contribution to the covariance of W_1 and W_2.

The covariance of $R_{1,j}^2$ and $R_{2,j}^2$ is

$$Cov\{R_{1,j}^2, R_{2,j}^2\} = Cov\{X_{1,j}^2, X_{2,j}^2\} + Cov\{X_{1,j}^2, Y_{2,j}^2\} + Cov\{Y_{1,j}^2, X_{2,j}^2\} + Cov\{Y_{1,j}^2, Y_{2,j}^2\} \quad (17)$$

Applying (11) to the pairs $(X_{1,j}, X_{2,j})$, $(X_{1,j}, Y_{2,j})$, $(Y_{1,j}, X_{2,j})$, and $(Y_{1,j}, Y_{2,j})$, and using (16) and (17)

$$\begin{aligned} Cov\{W_1, W_2\} = \sum_{j=1}^{n_0} \Big[& 2\nu_{1,j}^2 (\sigma_{X_{1,j}}^2 \sigma_{X_{2,j}}^2 + \sigma_{Y_{1,j}}^2 \sigma_{Y_{2,j}}^2) \\ & + 4\nu_{1,j} (m_{X_{1,j}} m_{X_{2,j}} \sigma_{X_{1,j}} \sigma_{X_{2,j}} + m_{Y_{1,j}} m_{Y_{2,j}} \sigma_{Y_{1,j}} \sigma_{Y_{2,j}}) \\ & + 4\nu_{2,j} (m_{X_{1,j}} m_{Y_{2,j}} \sigma_{X_{1,j}} \sigma_{Y_{2,j}} - m_{Y_{1,j}} m_{X_{2,j}} \sigma_{Y_{1,j}} \sigma_{X_{2,j}}) \\ & + 2\nu_{2,j}^2 (\sigma_{X_{1,j}}^2 \sigma_{Y_{2,j}}^2 + \sigma_{Y_{1,j}}^2 \sigma_{X_{2,j}}^2) \Big] \end{aligned} \quad (18)$$

By definition, the correlation coefficient of W_1 and W_2 is

$$\delta_W = \frac{Cov\{W_1, W_2\}}{\sqrt{Var\{W_1\} Var\{W_2\}}} \quad (19)$$

The substitution of (15) and (18) into (19) provides the power correlation coefficient of the general model described in Section II. Owing to the generality of such a model, its power correlation coefficient depends on a great number of free parameters. For this reason, in the following subsections, we cope with some cases that, despite being simplifications of the present one, are rather general and used in the literature. The assumptions of each case will enable us to express the power correlation coefficient in a simple and compact way.

C. One Cluster

In case each signal is composed of one *shared* cluster and no *unshared* cluster ($n_0 = n_1 = n_2 = 1$), the sums in (15) and (18) have only one term each. Thus, replacing these terms into (19) and after some algebraic manipulations, it results that

$$\begin{aligned} \delta_W = & \prod_{i=1}^{2} \left[\left[\eta_i^2 (2k_{X_i} + 1) + 2k_{Y_i} + 1 \right]^{-1/2} \right] \\ & \times \Big[2\nu_1 \left(\eta_1 \eta_2 \sqrt{k_{X_1} k_{X_2}} + \sqrt{k_{Y_1} k_{Y_2}} \right) \\ & + 2\nu_2 \left(\eta_1 \sqrt{k_{X_1} k_{Y_2}} - \eta_2 \sqrt{k_{Y_1} k_{X_2}} \right) \\ & + \nu_1^2 (1 + \eta_1 \eta_2) + \nu_2^2 (\eta_1 + \eta_2) \Big] \end{aligned} \quad (20a)$$

where

$$k_{X_i} = m_{X_i}^2 / \sigma_{X_i}^2 \qquad i = 1, 2 \quad (20b)$$

$$k_{Y_i} = m_{Y_i}^2 / \sigma_{Y_i}^2 \qquad i = 1, 2 \quad (20c)$$

$$\eta_i = \sigma_{X_i}^2 / \sigma_{Y_i}^2 \qquad i = 1, 2 \quad (20d)$$

Some models which consider the signal as composed of one cluster are listed below.

1) Rayleigh Model: In each signal of this model, the in-phase X_i and quadrature Y_i components have zero means and equal variances

$$m_{X_i} = m_{Y_i} = 0 \qquad i = 1, 2 \quad (21a)$$

$$\sigma_{X_i} = \sigma_{Y_i} = \sigma_i \qquad i = 1, 2 \quad (21b)$$

For this model, the power correlation coefficient is expressed simply as

$$\delta_W = \rho^2 \quad (22)$$

It has been observed in the literature [10], [11] that the envelope correlation coefficient of the Rayleigh model is very close to ρ^2, which corresponds exactly to the power correlation coefficient of such a model.

2) Ricean Model: In each signal of this model, the in-phase X_i and quadrature Y_i components have equal variances

$$\sigma_{X_i} = \sigma_{Y_i} = \sigma_i \qquad i = 1, 2 \quad (23)$$

For this model, the power correlation coefficient reduces to

$$\delta_W = \frac{\rho^2 + 2\rho\sqrt{k_1 k_2} \cos(\phi + \varphi_1 - \varphi_2)}{\sqrt{(1 + 2k_1)(1 + 2k_2)}} \quad (24a)$$

where[2]

$$\rho = \sqrt{\nu_1^2 + \nu_2^2} \quad (24b)$$

$$\phi = \arg(\nu_1 + I\nu_2) \quad (24c)$$

$$k_i = \frac{m_{X_i}^2 + m_{Y_i}^2}{2\sigma_i^2} \qquad i = 1, 2 \quad (24d)$$

$$\varphi_i = \arg(m_{X_i} + I m_{Y_i}) \qquad i = 1, 2 \quad (24e)$$

The parameter k_i represents the power ratio of the line-of-sight (or direct) component to the scattered component and is commonly referred to as the Ricean factor.

For $k_1 = k_2$, (24) equals the power correlation coefficient provided in [12]. In that work, it has been shown that the statistic in question constitutes an accurate approximation to the envelope correlation coefficient.

3) Hoyt Model: In this model, the means of the in-phase X_i and quadrature Y_i components are null

$$m_{X_i} = m_{Y_i} = 0 \qquad i = 1, 2 \quad (25)$$

In such a case, the power correlation coefficient simplifies to

$$\delta_W = \frac{\nu_1^2 (1 + \eta_1 \eta_2) + \nu_2^2 (\eta_1 + \eta_2)}{\sqrt{(1 + \eta_1^2)(1 + \eta_2^2)}} \quad (26)$$

The Hoyt model is symmetric for $0 \leq \eta_i \leq 1$ and $1 \leq \eta_i \leq \infty$ ($0 \leq \eta_i^{-1} \leq 1$), $i = 1, 2$. For (26), this can be proved replacing η_1 by η_1^{-1} and η_2 by η_2^{-1} and verifying that the expression of δ_W remains unchanged. Therefore, in the Hoyt model, the use of the range $0 \leq \eta_i \leq 1$ is sufficient to describe completely the influence of η_i on δ_W.

4) Asymmetrical $\eta - \kappa$ Model: In this model [5], the mean of one quadrature component is null. Thus, there are two Asymmetrical $\eta - \kappa$ models: one in which

$$m_{X_i} = 0 \qquad i = 1, 2 \quad (27a)$$

$$\kappa_i = \frac{m_{Y_i}^2}{\sigma_{X_i}^2 + \sigma_{Y_i}^2} = \frac{k_{Y_i}}{1 + \eta_i} \qquad i = 1, 2 \quad (27b)$$

[2] In this work, the function $arg(\cdot)$ is the argument of the complex number enclosed within, and I is the imaginary unit.

and an other in which

$$m_{Y_i} = 0 \qquad i = 1, 2 \qquad (27c)$$

$$\kappa_i = \frac{m_{X_i}^2}{\sigma_{X_i}^2 + \sigma_{Y_i}^2} = \frac{k_{X_i}}{1 + \eta_i^{-1}} \qquad i = 1, 2 \qquad (27d)$$

For the former case

$$\begin{aligned} \delta_W = &\prod_{i=1}^{2} \left[\left[2(1+\eta_i)\kappa_i + \eta_i^2 + 1 \right]^{-1/2} \right] \\ &\times \left[2\nu_1 \sqrt{(1+\eta_1)(1+\eta_2)\kappa_1\kappa_2} \right. \\ &\left. + \nu_1^2 \left(1 + \eta_1\eta_2\right) + \nu_2^2 \left(\eta_1 + \eta_2\right) \right] \end{aligned} \qquad (28a)$$

and for the latter case

$$\begin{aligned} \delta_W = &\prod_{i=1}^{2} \left[[2\eta_i(1+\eta_i)\kappa_i + \eta_i^2 + 1]^{-1/2} \right] \\ &\times \left[2\nu_1 \sqrt{\eta_1\eta_2(1+\eta_1)(1+\eta_2)\kappa_1\kappa_2} \right. \\ &\left. + \nu_1^2 \left(1 + \eta_1\eta_2\right) + \nu_2^2 \left(\eta_1 + \eta_2\right) \right] \end{aligned} \qquad (28b)$$

5) Symmetrical $\eta - \kappa$ Model: In this model [6], it is assumed that

$$\frac{m_{X_i}^2}{m_{Y_i}^2} = \frac{\sigma_{X_i}^2}{\sigma_{Y_i}^2} \qquad i = 1, 2 \qquad (29a)$$

and κ_i is defined as

$$\kappa_i = \frac{m_{X_i}^2}{\sigma_{X_i}^2} = \frac{m_{Y_i}^2}{\sigma_{Y_i}^2} \qquad i = 1, 2 \qquad (29b)$$

Thus, for this model

$$\begin{aligned} \delta_W = &\prod_{i=1}^{2} \left[\left[(1+\eta_i^2)(1+2\kappa_i) \right]^{-1/2} \right] \\ &\times [2\sqrt{\kappa_1\kappa_2}[\nu_1 \left(1 + \eta_1\eta_2\right) + \nu_2 \left(\eta_1 - \eta_2\right)] \\ &+ \nu_1^2 \left(1 + \eta_1\eta_2\right) + \nu_2^2 \left(\eta_1 + \eta_2\right)] \end{aligned} \qquad (30)$$

6) Generalized $\eta - \kappa$ Model: In this model, the in-phase and quadrature components have arbitrary means and variances. Thus, the power correlation coefficient of this model is obtained directly by changing the notation from k_{X_i} and k_{Y_i} to κ_{X_i} and κ_{Y_i} in the equation (20).

D. Identically Distributed Clusters (I.D. Clusters)

In this subsection, we will investigate the case in which the marginal statistics are identical for all clusters

$$m_{X_{i,j}} = m_{X_i} \qquad m_{Y_{i,j}} = m_{Y_i} \qquad j = 1, 2, ..., n_i \qquad (31a)$$

$$\sigma_{X_{i,j}} = \sigma_{X_i} \qquad \sigma_{Y_{i,j}} = \sigma_{Y_i} \qquad j = 1, 2, ..., n_i \qquad (31b)$$

and the joint statistics are equal for all *shared* clusters

$$\nu_{1,j} = \nu_1 \qquad \nu_{2,j} = \nu_2 \qquad j = 1, 2, ..., n_0 \qquad (31c)$$

Thus, substituting (15) and (18) into (19), considering (31), and making some algebraic manipulations, the power correlation coefficient equals

$$\begin{aligned} \delta_W = &\sqrt{\frac{\mu_1}{\mu_2}} \alpha \prod_{i=1}^{2} \left[\left[\eta_i^2 (2k_{X_i} + 1) + 2k_{Y_i} + 1 \right]^{-1/2} \right] \\ &\times \left[2\nu_1 \left(\eta_1\eta_2 \sqrt{k_{X_1}k_{X_2}} + \sqrt{k_{Y_1}k_{Y_2}} \right) \right. \\ &+ 2\nu_2 \left(\eta_1 \sqrt{k_{X_1}k_{Y_2}} - \eta_2 \sqrt{k_{Y_1}k_{X_2}} \right) \\ &\left. + \nu_1^2 \left(1 + \eta_1\eta_2\right) + \nu_2^2 \left(\eta_1 + \eta_2\right) \right] \end{aligned} \qquad (32a)$$

where k_{X_i}, k_{Y_i}, and η_i are given in (20) and

$$\alpha = \frac{\mu_0}{\mu_1} \qquad (32b)$$

$$\mu_i = n_i \qquad i = 0, 1, 2 \qquad (32c)$$

The parameter α denotes the proportion of clusters of S_1 that are *shared* with S_2. For $\mu_0 = \mu_1 = \mu_2 = 1$, (32) reduces to (20).

It is noteworthy that, in (15) and (18), n_0, n_1, and n_2 are necessarily integers. However, differently from the general case, there is no mathematical constraint in considering μ_0, μ_1, and μ_2 as non-integers in (32). In fact, the extension of those parameters from the discrete domain (n_i) to the continuous domain (μ_i) has been broadly applied in the literature [7]–[9].

Two models which consider i.d. clusters are the Nakagami-m [7] and the $\eta - \mu$ [9] ones. In the $\kappa - \mu$ model [8], the relation (31a) is not satisfied, and hence the clusters are non-identically distributed (n.d.). Here, we shall deal with the $\kappa - \mu$ model considering i.d. clusters; for n.d. clusters, the power correlation coefficient of that model can be obtained through the general result provided in the Subsection III-B.

1) Nakagami-m Model: In this model [7], in addition to i.d. clusters, the in-phase $X_{i,j}$ and quadrature $Y_{i,j}$ components of each cluster have zero means and equal variances

$$m_{X_i} = m_{Y_i} = 0 \qquad i = 1, 2 \qquad (33a)$$

$$\sigma_{X_i} = \sigma_{Y_i} = \sigma_i \qquad i = 1, 2 \qquad (33b)$$

With these considerations, (32) reduces to

$$\delta_W = \sqrt{\frac{\mu_1}{\mu_2}} \alpha \rho^2 \qquad (34)$$

where ρ is given in (24b). For $\mu_0 = \mu_1 = \mu_2 = 1$, (34) simplifies to (22); for $\alpha = 1$, (34) equals the power correlation coefficient provided in [13]. In this case, that statistic constitutes an accurate approximation to the envelope correlation coefficient [14].

2) $\kappa - \mu$ Model (I.D. Clusters): Generically, this model assumes that [8]

$$\sigma_{X_i} = \sigma_{Y_i} = \sigma_i \qquad i = 1, 2 \qquad (35a)$$

and defines the parameter κ_i as [8]

$$\kappa_i = \frac{\sum_{j=1}^{n_i} \left[m_{X_{i,j}}^2 + m_{Y_{i,j}}^2 \right]}{2n_i\sigma_i^2} \qquad i = 1, 2 \qquad (35b)$$

Considering i.d. clusters, the parameter κ_i equals

$$\kappa_i = \frac{m_{X_i}^2 + m_{Y_i}^2}{2\sigma_i^2} \qquad i = 1, 2 \qquad (36)$$

and the power correlation coefficient is

$$\delta_W = \sqrt{\frac{\mu_1}{\mu_2}} \frac{\alpha\left[\rho^2 + 2\rho\sqrt{\kappa_1\kappa_2}\cos\left(\phi + \varphi_1 - \varphi_2\right)\right]}{\sqrt{(1+2\kappa_1)(1+2\kappa_2)}} \tag{37}$$

where ρ, ϕ, and φ_i are given in (24). For $\mu_0 = \mu_1 = \mu_2 = 1$, (37) simplifies to (24a).

3) $\eta - \mu$ Model: In this model [9], in addition to i.d. clusters, the in-phase $X_{i,j}$ and quadrature $Y_{i,j}$ components of each cluster have zero means

$$m_{X_i} = m_{Y_i} = 0 \qquad i = 1, 2 \tag{38}$$

In this case, the power correlation coefficient is

$$\delta_W = \sqrt{\frac{\mu_1}{\mu_2}} \frac{\alpha\left[\nu_1^2(1+\eta_1\eta_2) + \nu_2^2(\eta_1+\eta_2)\right]}{\sqrt{(1+\eta_1^2)(1+\eta_2^2)}} \tag{39}$$

where ρ is given in (24b). For $\mu_0 = \mu_1 = \mu_2 = 1$, (39) simplifies to (26).

E. Stationary Environments

For stationary environments, the following equalities must be substituted into the expressions of the power correlation coefficients of their respective models

- Rice: $k_1 = k_2$ and $\varphi_1 = \varphi_2$;
- Hoyt: $\eta_1 = \eta_2$;
- Asymmetrical $\eta - \kappa$: $\eta_1 = \eta_2$ and $\kappa_1 = \kappa_2$;
- Symmetrical $\eta - \kappa$: $\eta_1 = \eta_2$ and $\kappa_1 = \kappa_2$;
- Generalized $\eta - \kappa$: $\eta_1 = \eta_2$, $\kappa_{X_1} = \kappa_{X_2}$, and $\kappa_{Y_1} = \kappa_{Y_2}$;
- Nakagami-m: $\mu_1 = \mu_2$;
- $\kappa - \mu$: $\mu_1 = \mu_2$, $\kappa_1 = \kappa_2$, and $\varphi_1 = \varphi_2$;
- $\eta - \mu$: $\mu_1 = \mu_2$ and $\eta_1 = \eta_2$.

IV. Numerical Results and Comments

In this section, we present some plots illustrating the power correlation coefficient of the Rayleigh, Rice, and Hoyt distributions in stationary environments. For such environments

$$\delta_W = \frac{\rho^2 + 2k\nu_1}{1+2k} \qquad \text{Ricean Model} \tag{40a}$$

$$\delta_W = \frac{\nu_1^2(1+\eta^2) + 2\nu_2^2\eta}{1+\eta^2} \qquad \text{Hoyt Model} \tag{40b}$$

In the specification of the parameters ν_1 and ν_2, we shall assume the mathematical model described by Jakes [10], which provides

$$\nu_1 = \frac{E\left\{D(\Theta)\cos\left[\beta d\cos(\Theta) - \Delta\omega \mathrm{T}\right]\right\}}{E\{D(\Theta)\}} \tag{41a}$$

$$\nu_2 = \frac{E\left\{D(\Theta)\sin\left[\beta d\cos(\Theta) - \Delta\omega \mathrm{T}\right]\right\}}{E\{D(\Theta)\}} \tag{41b}$$

where β is the phase constant, d is the distance between the reception points, $\Delta\omega$ is the angular frequency difference between the transmitted signals, and Θ and T are random variables which denote, respectively, the angles of arrival and the times of arrival of the scattered waves.

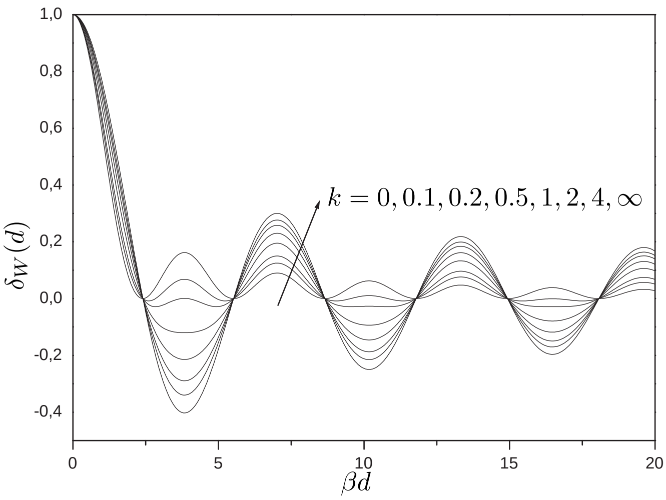

Fig. 1. Space correlation coefficient of the instantaneous power for the Ricean model with $k_1 = k_2$ and $\varphi_1 = \varphi_2$.

With the intention of maintaining compatibility with the results already available for the Rayleigh case [10], we shall consider

$$D(\theta) = 1 \tag{42a}$$

$$p_{\Theta,\mathrm{T}}(\theta, \mathrm{t}) = p_\Theta(\theta) p_\mathrm{T}(\mathrm{t}) \tag{42b}$$

$$p_\Theta = \frac{1}{2\pi} \qquad 0 \le \theta \le 2\pi \tag{42c}$$

$$p_\mathrm{T}(\mathrm{t}) = \frac{1}{\overline{\overline{\mathrm{T}}}} \exp\left(-\frac{\mathrm{t}}{\overline{\overline{\mathrm{T}}}}\right) \qquad \mathrm{t} \ge 0 \tag{42d}$$

where $\overline{\mathrm{T}}$ is the time delay spread. From (41) and (42)

$$\nu_1 = \frac{J_0(\beta d)}{1 + (\Delta\omega\overline{\mathrm{T}})^2} \tag{43a}$$

$$\nu_2 = -\frac{\Delta\omega\overline{\mathrm{T}} J_0(\beta d)}{1 + (\Delta\omega\overline{\mathrm{T}})^2} \tag{43b}$$

The space correlation coefficient $\delta_W(d)$ and the frequency correlation coefficient $\delta_W(\Delta\omega)$ are obtained by setting, respectively, $\Delta\omega = 0$ and $d = 0$. For a mobile receiver, the distance d is a function of the time τ, and hence $\delta_W(d)$ is converted into the time correlation coefficient $\delta_W(\tau)$.

A. Space (or Time) Correlation Coefficient In Stationary Environments

Fig. 1 shows some plots of $\delta_W(d)$ for the Ricean model with different values of k. As it can be seen, the space correlation coefficient is minimum at $k = 0$ (Rayleigh model) and increases with increasing k.

For the Hoyt model, (40b) and (43b) provide a space correlation coefficient independent of η: from $\Delta\omega = 0$ and (43b), $\nu_2 = 0$; hence, substituting $\nu_2 = 0$ into (40b), $\delta_W = \nu_1^2$ (independent of η).

B. Frequency Correlation Coefficient In Stationary Environments

Similarly to the Hoyt model in the space (or time) domain, the Ricean model with ν_1 and ν_2 as provided in (43) has a

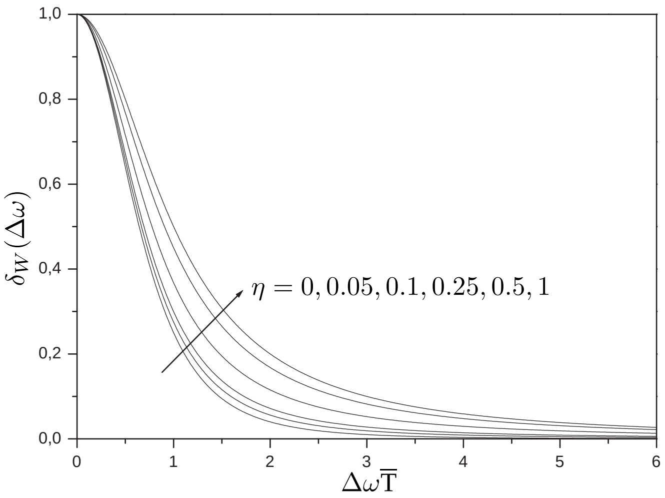

Fig. 2. Frequency correlation coefficient of the instantaneous power for the Hoyt model with $\eta_1 = \eta_2$.

frequency correlation coefficient independent of k: from $d = 0$, (24b), and (43), $\nu_1 = \rho^2$; hence, substituting $\nu_1 = \rho^2$ into (40a), $\delta_W = \rho^2$ (independent of k).

Fig. 2 illustrates some plots of $\delta_W(\Delta\omega)$ for the Hoyt model with different values of η. It can be observed that, in the range $0 \leq \eta \leq 1$, the greater the value of η the stronger the frequency correlation coefficient between the instantaneous powers. Therefore, since the Hoyt model is symmetric for $0 \leq \eta \leq 1$ and $1 \leq \eta \leq \infty$, the strongest frequency correlation coefficient occurs when $\eta = 1$ (Rayleigh model).

V. Conclusion

In this work, we have provided the power correlation coefficient of a very general fading model. Besides allowing for both identically and non-identically distributed clusters, and for both stationary and nonstationary environments, the model includes the Rayleigh, Rice, Nakagami-m, Hoyt, among other distributions as particular cases. Then, after deriving the power correlation coefficient of the general model, it has been presented simple and compact expressions of that statistic for special models. Finally, assuming stationary environments and the mathematical model proposed by Jakes [10], the Rayleigh, Ricean, and Hoyt power correlation coefficients have been contrasted in both space domain and frequency domain.

References

[1] R. H. Clarke, “A statistical theory of mobile-radio reception,” *Bell Systems Technical Journal*, vol. 47, pp. 957–1000, July–Aug. 1968.

[2] S. O. Rice, “Mathematical analysis of random noise,” *Bell Systems Technical Journal*, vol. 23, no. 3, pp. 282–332, July 1944.

[3] ——, “Mathematical analysis of random noise,” *Bell Systems Technical Journal*, vol. 24, no. 1, pp. 46–156, Jan. 1945.

[4] R. S. Hoyt, “Probability functions for the modulus and angle of the normal complex variate,” *Bell System Technical Journal*, vol. 26, pp. 318–359, Apr. 1947.

[5] M. D. Yacoub, G. Fraidenraich, H. B. Tercius, and F. C. Martins, “The Assymmetrical $\eta - \kappa$ distribution,” in *Proc. International Workshop on Telecommunications*, Aug. 2004, pp. 89–93.

[6] ——, “The Symmetrical $\eta - \kappa$ distribution,” in *Proc. IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, vol. 4, Sept. 2004, pp. 2426–2430.

[7] M. Nakagami, “The m-distribution - a general formula of intensity distribution of rapid fading,” in *Statistical Methods in Radio Wave Propagation*, W. C. Hoffman, Ed. Oxford, England: Pergamon, 1960, pp. 3–36.

[8] M. D. Yacoub, “The κ-μ distribution: a general fading distribution,” in *Proc. IEEE Vehicular Technology Conference*, vol. 3, Atlantic City, NJ USA, Oct. 2001, pp. 1427–1431.

[9] ——, “The η-μ distribution: a general fading distribution,” in *Proc. IEEE Vehicular Technology Conference*, vol. 2, Boston, MA USA, Sept. 2000, pp. 872–877.

[10] W. C. Jakes, *Microwave Mobile Communications*. New York: Willey, 1997.

[11] W. C. Y. Lee, *Mobile Communications Engineering*. McGraw-Hill, 1997.

[12] Y. Karasawa and H. Iwai, “Modeling of signal envelope correlation of line-of-sight fading with applications to frequecy correlation analysis,” *IEEE Transactions on Communications*, vol. 42, no. 6, June 1994.

[13] J. Reig, L. Rubio, and N. Cardona, “Bivariate Nakagami-m Distribution with Arbitrary Fading Parameters,” *Electronics Letters*, vol. 38, no. 25, pp. 1715–1717, Dec. 2002.

[14] J. C. S. Santos Filho, G. Fraidenraich, U. S. Dias, and M. D. Yacoub, “On the Nakagami-m Crosscorrelation Function,” in *Proc. International Microwave and Optoelectronics Conference*, 2005.